Foundation of Cryptography, Lecture 10
Multiparty Computation

Handout Mode

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January 26, 2017

Section 1

The Model

Multiparty Computation

- Multiparty Computation – computing a functionality f
- Secure Multiparty Computation: compute f in a “secure manner"
  - Correctness
  - Privacy
  - Independence of inputs
  - Guaranteed output delivery
  - Fairness : corrupted parties should get their output iff the honest parties do
  - and ...
- Examples: coin-tossing, broadcast, electronic voting, electronic auctions
- How should we model it?
- Real Vs. Ideal paradigm

Real-model execution

For a a pair of algorithms $\overline{A} = (A_1, A_2)$ and inputs $x_c, x_1, x_2 \in \{0,1\}^*$, let $\mathrm{REAL}_{\overline{A}}(x_c, x_1, x_2)$ be the joint output of $(A_1(x_c, x_1), A_2(x_c, x_2))$.

Given a two-party protocol π, an algorithm taking the role of one of the parties in π is:

- Malicious — acts arbitrarily.
- Honest — acts exactly according to π.
- Semi-honest — acts according to π, but might output **additional information**.

$\overline{A} = (A_1, A_2)$ is an admissible with respect to π, if at least one party is honest.

Ideal model execution

For a pair of oracle-aided algorithms $\overline{B} = (B_1, B_2)$, inputs $x_c, x_1, x_2 \in \{0,1\}^*$ and a function $f = (f_1, f_2)$, let $\text{IDEAL}^f_{\overline{B}}(x_c, x_1, x_2)$ be the joint output of the parties in the end of the following experiment:

1. The input of B_i is (x_c, x_i).
2. B_i sends value y_i to the trusted party .
3. Trusted party sends $z_i = f_i(y_0, y_1)$ to B_i in an arbitrary order.
4. Each party outputs some value.

The actual definition allows a party after receiving its output, to instruct f not to send the the output to the other party.

An oracle-aided algorithm B taking the role of one of the parties is:

- Malicious — acts arbitrarily.
- Honest — sends its private input to the trusted party (i.e., sets $y_i = x_i$), and its only output is the value it gets from the trusted party (i.e., z_i).
- Semi-honest, sends its input to the trusted party, outputs z_i plus possibly additional information.

$\overline{B} = (B_1, B_2)$ is admissible, if at least one party is honest.

Secure computation

Definition 1 (secure computation)

A protocol π securely computes f, if $\forall$ admissible PPT pair $\overline{A} = (A_1, A_2)$ for π, exists admissible oracle-aided PPT pair $\overline{B} = (B_1, B_2)$, s.t.

$$\{\text{REAL}_{\overline{A}}(x_c, x_1, x_2)\}_{x_c, x_1, x_2 \in \{0,1\}^*} \approx_c \{\text{IDEAL}^f_{\overline{B}}(x_c, x_1, x_2)\}_{x_c, x_1, x_2 \in \{0,1\}^*}$$

In case $\overline{A}$ is honest, we require that $\overline{B}$ is honest, and the ensembles to be identical.

- Recall that the enumeration index (i.e., x_c, x_1, x_2) is given to the distinguisher.
- π securely computes f implies that π computes f correctly.
- Security parameter
- Auxiliary inputs
- We focus on semi-honest adversaries.

Section 2

Oblivious Transfer

Oblivious transfer

An (one-out-of-two) OT protocol securely computes the functionality $\mathsf{OT} = (\mathsf{OT}_S, \mathsf{OT}_R))$ over $(\{0,1\}^* \times \{0,1\}^*) \times \{0,1\}$, where $\mathsf{OT}_S(\cdot) = \perp$ and $\mathsf{OT}_R((\sigma_0, \sigma_1), i) = \sigma_i$.

- “Complete" for multiparty computation
- We show how to construct for bit inputs.

Oblivious transfer from trapdoor permutations

Let (G, f, Inv) be a TDP and let b be an hardcore predicate for f.

Protocol 2 ((S, R))

Common input: 1^n
S's input: $\sigma_0, \sigma_1 \in \{0,1\}$.
R's input: $i \in \{0,1\}$.

1. S chooses $(e, d) \leftarrow G(1^n)$, and sends e to R.
2. R chooses $x_0, x_1 \leftarrow \{0,1\}^n$, sets $y_i = f_e(x_i)$ and $y_{1-i} = x_{1-i}$, and sends y_0, y_1 to S.
3. S sets $c_j = b(\mathrm{Inv}_d(y_j)) \oplus \sigma_j$, for $j \in \{0,1\}$, and sends (c_0, c_1) to R.
4. R outputs $c_i \oplus b(x_i)$.

Claim 3

Protocol 2 securely computes OT (in the semi-honest model).

Proving Claim 3

We need to prove that $\forall$ semi-honest admissible PPT pair $\overline{A} = (A_1, A_2)$ for (S, R), exists admissible oracle-aided PPT pair $\overline{B} = (B_1, B_2)$ s.t.

$$\{\text{REAL}_{\overline{A}}(1^n, (\sigma_0, \sigma_1), i)\} \approx_c \{\text{IDEAL}^{\text{OT}}_{\overline{B}}(1^n, (\sigma_0, \sigma_1), i)\}, \tag{1}$$

where the enumeration is over $n \in \mathbb{N}$ and $\sigma_0, \sigma_1, i \in \{0, 1\}$.

R's security

For a semi-honest implementation S' of S, define the oracle-aided semi-honest strategy $S'_{\mathcal{I}}$ as follows.

Algorithm 4 ($S'_{\mathcal{I}}$)

input: $1^n, \sigma_0, \sigma_1$

1. Send (σ_0, σ_1) to the trusted party.
2. Emulate $(S'(1^n, \sigma_0, \sigma_1), R(1^n, 0))$.
3. Output the output that S' does.

Let $\overline{A} = (S', R)$ and $\overline{B} = (S'_{\mathcal{I}}, R_{\mathcal{I}})$, where $R_{\mathcal{I}}$ is honest.

Claim 5

$\{\mathrm{REAL}_{\overline{A}}(1^n, (\sigma_0, \sigma_1), i)\} \equiv \{\mathrm{IDEAL}^{\mathrm{OT}}_{\overline{B}}(1^n, (\sigma_0, \sigma_1), i)\}$.

Proof?

S's security

For a semi-honest implementation R' of R, define the oracle-aided semi-honest strategy $R'_{\mathcal{I}}$ as follows.

Algorithm 6 ($R'_{\mathcal{I}}$)

input: $1^n, i \in \{0,1\}$,

1. Send i to the trusted party, and let σ be its answer.
2. Emulate $(S(1^n, \sigma_0, \sigma_1), R'(1^n, i))$, for $\sigma_i = \sigma$ and $\sigma_{1-i} = 0$.
3. Output the output that R' does.

Let $\overline{A} = (S, R')$ and $\overline{B} = (S_{\mathcal{I}}, R'_{\mathcal{I}})$, where $S_{\mathcal{I}}$ is honest.

Claim 7

$\{\mathrm{REAL}_{\overline{A}}(1^n, (\sigma_0, \sigma_1), i)\} \approx_c \{\mathrm{IDEAL}^{\mathrm{OT}}_{\overline{B}}(1^n, (\sigma_0, \sigma_1), i)\}$.

Proof?

Section 3

Yao Garbled Circuit

Before we start

- Fix a (multiple message) semantically-secure private-key encryption scheme (G, E, D) with
  1. $G(1^n) = U_n$.
  2. For any $m \in \{0,1\}^*$
     $\Pr_{d,d' \leftarrow \{0,1\}^n}[D_d(E_{d'}(m)) \neq \perp] = \text{neg}(n)$.

  Can we construct such a scheme?

  append 0^n at the end of the message...

- Boolean circuits: gates, wires, inputs, outputs, values, computation

The Garbled Circuit

Fix a Boolean circuit C and $n \in \mathbb{N}$.

- Let $\mathcal{W}$ and $\mathcal{G}$ be the (indices) of **wires** and **gates** of C, respectively.
- For $w \in \mathcal{W}$, associate a pair of random 'keys" $k_w = (k_w^0, k_w^1) \in (\{0,1\}^n)^2$.
- For $g \in \mathcal{G}$ with input wires i and j, and output wire h, let $T(g)$ be the following table:

input wire i	input wire j	output wire h	hidden output wire
k_i^0	k_j^0	$k_h^{g(0,0)}$	$E_{k_i^0}(E_{k_j^0}(k_h^{g(0,0)}))$
k_i^0	k_j^1	$k_h^{g(0,1)}$	$E_{k_i^0}(E_{k_j^1}(k_h^{g(0,1)}))$
k_i^1	k_j^0	$k_h^{g(1,0)}$	$E_{k_i^1}(E_{k_j^0}(k_h^{g(1,0)}))$
k_i^1	k_j^1	$k_h^{g(1,1)}$	$E_{k_i^1}(E_{k_j^1}(k_h^{g(1,1)}))$

Figure: Table for gate g, with input wires i and j, and output wire h.

The Garbled Circuit, cont.

input wire i	input wire j	output wire h	hidden output wire
k_i^0	k_j^0	$k_h^{g(0,0)}$	$E_{k_i^0}(E_{k_j^0}(k_h^{g(0,0)}))$
k_i^0	k_j^1	$k_h^{g(0,1)}$	$E_{k_i^0}(E_{k_j^1}(k_h^{g(0,1)}))$
k_i^1	k_j^0	$k_h^{g(1,0)}$	$E_{k_i^1}(E_{k_j^0}(k_h^{g(1,0)}))$
k_i^1	k_j^1	$k_h^{g(1,1)}$	$E_{k_i^1}(E_{k_j^1}(k_h^{g(1,1)}))$

Let $\mathcal{I}$ and $\mathcal{O}$ be the input and outputs wires of C.

- For $g \in \mathcal{G}$, let $\widetilde{T}(g)$ be a random permutation of the fourth column of $T(g)$.
- For $w \in \mathcal{W}$, let $C(x)_w$ be the **bit-value** computation of $C(x)$ assigns to w
- Given
  1. $\widetilde{T} = \{(g, \widetilde{T}(g))\}_{g \in \mathcal{G}}$.
  2. $\{k_w^{C(x)_w}\}_{w \in \mathcal{I}}$ for some x.
  3. $\{(w, k_w = (k_w^0, k_w^1)\}_{w \in \mathcal{O}}$.

  One can efficiently compute $C(x)$.
- (essentially) The above leaks no additional information about x!

The protocol

- Let $f(x_A, x_B) = (f_A(x_A, x_B), f_B(x_A, x_B))$ be a function, and let C be a circuit that computes f.
- Let $\mathcal{I}_A$ and $\mathcal{I}_B$ be the input wires corresponds to x_A and x_B respectively in C, and let $\mathcal{O}_A$ and $\mathcal{O}_B$ be the output wires corresponds to f_A and f_B outputs respectively in C.
- Recall that $C(x)_w$ is the bit-value the computation of $C(x)$ assigns to w.
- Let (S, R) be a secure protocol for OT.

Protocol 8 ((A, B))

Common input: 1^n. **A/B's input:** x_A / x_B

1. A samples at random $\{k_w = (k_w^0, k_w^1)\}_{w \in \mathcal{W}}$, and generate $\widetilde{T}$.
2. A sends $\widetilde{T}$ and $\{(w, k_w^{C(x_1,\cdot)_w})\}_{w \in \mathcal{I}_A}$ to B.
3. $\forall w \in \mathcal{I}_B$, A and B interact in $(S(k_w), R(C(\cdot, x_2)_w))(1^n)$.
4. B computes the (garbled) circuit, and sends $\{(w, k_w^{C(x_1,x_2)_w})\}_{w \in \mathcal{O}_A}$ to A.
5. A sends $\{(w, k_w)\}_{w \in \mathcal{O}_B}$ to B.
6. The parties compute $f_A(x_1, x_2)$ and $f_B(x_1, x_2)$ respectively.

Example, computing OR

On board...

Claim 9

Protocol 8 securely computes f (in the semi-honest model)

Proof: We focus on the security of A. For a semi-honest B', define

Algorithm 10 ($B'_{\mathcal{I}}$)

input: 1^n and x_B.

1. Send x_B to the trusted party, and let o_B be its answer.
2. Emulate the first 4 steps of $(A(1^{|x_A|}), B'(x_B)(1^n))$.
3. For each $w \in \mathcal{O}_B$: permute the order of the pair k_w according to o_B, and the key of w computed in the emulation.
4. Complete the emulation, and output the output that B' does.

Claim: $B'_{\mathcal{I}}$ is a good "simulator" for B'.

Security of A ?

Extensions

- Efficiently computable f

  Both parties first compute C_f – a circuit that compute f for inputs of the right length

- Hiding C? All but its size

Malicious model

The parties prove that they act “honestly":

1. Forces the parties to chose their random coin properly
2. Before each step, the parties prove in $\mathcal{ZK}$ that they followed the prescribed protocol (with respect to the random-coins chosen above)

Course summary

See diagram

What we did not cover

- “Few" reductions
- Environment security (e.g., UC)
- Information theoretic crypto
- Non-generic constructions : number theory, lattices
- Impossibility results
- “Real life cryptography" (e.g., Random oracle model)
- Security
- Differential privacy
  (maybe it is still not too late to register to BarIlan winter School...)
- and....

Advanced course (next semester, same time)

- Cryptography in low depth
- Impossibility result
- Computation notion of entropy and their applications
- and more...

Students seminar on MPC, Tuesdays 10–12

- Will cover (some of) the basic topics in this exciting area.
- Lectures list to be publish early February.
- Each students gives a two hours (white board) talk.
- A graduate seminar, but undergrad who took this course are welcome.

The exam